**BOOLEAN ALGEBRA SOLUTION FOR ASSIGNMENT**
**Govt. Mizo HSS**

6. State the principal of duality in boolean algebra and give the dual of the boolean expression.

$(X+Y).(\overline{X}+\overline{Z}).(Y+Z)$

According to the principle of duality, a true boolean statement or expression can be converted to its dual form by replacing the "0"s in the statement or expression with "1"s, and vice versa and by replacing the "+"s in the statement or expression with "."s, and vice versa.

The dual form of $(X+Y)(\overline{X}+\overline{Z}).(Y+Z)$ is $(X.Y)+(\overline{X}.\overline{Z})+(Y.Z)$

In the above dual expression all the "+"s were replaced by "."s, and vice versa.

7. Prove the idempotence law of boolean algebra with the help of a truth table.

| X | X | Output X.X | X+X |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 |

8. Use the duality theorem to derive another boolean relation from $A+\overline{A}B = A+B$

The duality theorem states that the AND operator is replaced with the OR operator. And OR operator is replaced with AND operator. Binary number 0 is replaced with binary number 1. And Binary number 1 is replaced with binary number 0.

Applying duality principle:

$A+\overline{A}B = A+B$

$A.(\overline{A}+B) = AB$ (Apply Duality & Distributive law)

$A\overline{A}+AB = AB$ (Apply Complimentary law)

$0+AB = AB$ (Properties of 0 or Identity law)

$AB = AB$

$\therefore$ LHS = RHS

9. What would be the complement of the following?

**(a) $\overline{A}(B\overline{C}+\overline{B}C)$**

*Complement the expression*

$= \overline{\overline{A}(B\overline{C}+\overline{B}C)}$ *Apply Demorgan's first law*

$= \overline{\overline{A}}\ (\overline{(B\overline{C})}\ \overline{(\overline{B}C)})$ *Apply Demorgan's second law (adding + inside and outside( ) )*

$= \overline{\overline{A}}+((\overline{B}+\overline{\overline{C}})\ (\overline{\overline{B}}+\overline{C}))$ *Apply Involution law*

$= A+((\overline{B}+C)\ (B+\overline{C}))$ *Apply Distribution law*

$= A+(\ (B+\overline{C})\overline{B}+(B+\overline{C})C\ )$ *Apply Distribution law*

$= A+\ (B\overline{B}+\overline{B}\overline{C}+CB+C\overline{C})$ *Apply Complementary law*

$= A+(0+\overline{B}\overline{C}+CB+0)$ *Apply Identity law*

$= A+\overline{B}\overline{C}+CB$

**(b)** **$xy + \bar{y}z + \bar{z}z$**

*Complement the expression*

$= \overline{(xy + \bar{y}z + \bar{z}z)}$ *First law of Demorgan*

$= \overline{(xy)}\ \overline{(\bar{y}z)}\ \overline{(\bar{z}z)}$ *Second Law of Demorgan*

$= (\bar{x} + \bar{y})\ (\bar{\bar{y}} + \bar{z})(\ \bar{\bar{z}} + \bar{z})$ *Apply Involution law*

$= \mathbf{(\bar{x} + \bar{y})(y + \bar{z})(z + \bar{z})}$ *Apply Complimentary law*

$= (\bar{x}+\bar{y})(y+\bar{z})1$ *Apply Identity law*

$= (\bar{x}+\bar{y})(y+\bar{z})$ *Apply Distribution law*

$= \bar{x}y+\bar{x}\ \bar{z}+\bar{y}y+\bar{y}\ \bar{z}$ *Apply Indempotent law*

$= \bar{x}y+\bar{x}\ \bar{z}+0+\bar{y}\ \bar{z}$ *Apply Identity law*

$= \bar{x}y+\bar{x}\ \bar{z}+\bar{y}\ \bar{z}$ *Apply Consensus Law*

$= \bar{x}y+\bar{y}\ \bar{z}$

10. Find the complement of the following Boolean function: F1 = $A\bar{B} + \bar{C}\bar{D}$

The Complement the expression:

$= \overline{A\bar{B}\ +\ \bar{C}\bar{D}}$ *Apply Demorgan's first law*

$= \overline{(A\bar{B})}.\ \overline{(\bar{C}\bar{D})}$ *Apply DeMorgan's Second Law*

$= (\bar{A} + \bar{\bar{B}}).\ (\bar{\bar{C}} + \bar{\bar{D}})$ *Apply Involution law*

$= (\bar{A}+ B)\ (C+D)$ *Apply Distribution law*

$= (C+D)\bar{A}+(C+D)B$ *Apply Distribution law*

$= \bar{A}C+\bar{A}D+BC+BD$

11. Find the complement of Boolean expression $(A+\bar{B}+C)\ (A+\bar{B}C)$

The Complement the expression:

$= \overline{(A + \bar{B} + C)\ (A + \bar{B}C)}$ Apply Demorgan's second law

$= \overline{(A + \bar{B} + C)} + \overline{(A + \bar{B}C)}$ Apply Demorgan's first law

$= (\bar{A}.\bar{\bar{B}}.\bar{C}) + (\ \bar{A}.(\bar{\bar{B}}+\bar{C}))$ Apply Involution law

$= (\bar{A}B\bar{C})+(\bar{A}(B+\bar{C})$ Apply Distribution law

$= \bar{A}B\bar{C}+\bar{A}B+\bar{A}\bar{C}$ Apply Absorption law (A+AB=A)

$= \bar{A}B+\bar{A}\bar{C}$

12. Find the complement of Boolean expression $\bar{A}D+\bar{C}D+\bar{A}B$

The complement of the expression:

$= \overline{\bar{A}D + \bar{C}D + \bar{A}B}$ *Apply Demorgan's first law*

$= \overline{(\bar{A}D)}.\overline{(\bar{C}D)}.\overline{(\bar{A}B)}$ *Apply Demorgan's second law*

$= (\bar{\bar{A}}+\bar{D}).(\bar{\bar{C}}+\bar{D}).(\bar{\bar{A}}+\bar{B})$ *Apply Involution law*

$= \mathbf{(A+\bar{D})(C+\bar{D})(A+\bar{B})}$ *Apply Distribution law*

$= (AC+A\bar{D}+\bar{D}C+\bar{D}\bar{D})(A+\bar{B})$ *Apply Distribution law*

$= (AC+\bar{D})\ (A+\bar{B})$ *Apply Distribution law*

$= A\mathbf{AC}+\mathbf{AC\bar{B}}+A\bar{D}+\bar{D}\bar{B}$ *Apply Indempotence law (italic) & Absorption law (in **bold**)*

$= AC+A\bar{D}+\bar{D}\bar{B}$

*Solved by* Sir **Mawia HL**